On Proportional 2-Choosability of Graphs with a Bounded Palette

Paul Shin

College of Lake County

October 14, 2019

Joint work with Jeffrey Mudrock, Robert Piechota, and Tim Wagstrom

Classical Coloring

- A ***proper*** k***-coloring*** of a graph G is a labeling $f : V(G) \to S$, where $|S| = k$ and $f(u) \neq f(v)$ whenever u and v are adjacent in G.

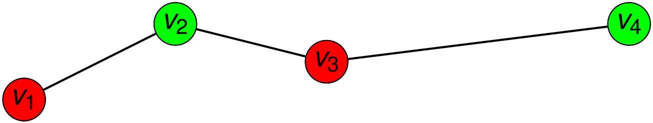
Classical Coloring

- A ***proper*** k***-coloring*** of a graph G is a labeling $f : V(G) \to S$, where $|S| = k$ and $f(u) \neq f(v)$ whenever u and v are adjacent in G.

- For a color $c \in S$, the ***color class*** of c, denoted by $f^{-1}(c)$, is the set of vertices to which f assigns the color c.

Classical Coloring

- A ***proper*** k***-coloring*** of a graph G is a labeling $f : V(G) \to S$, where $|S| = k$ and $f(u) \neq f(v)$ whenever u and v are adjacent in G.
- For a color $c \in S$, the ***color class*** of c, denoted by $f^{-1}(c)$, is the set of vertices to which f assigns the color c.
- Note that the color classes are independent sets.



Equitable Coloring

- An ***equitable*** k***-coloring*** of a graph G is a proper k-coloring of G such that the sizes of the color classes differ by at most one.

Equitable Coloring

- An ***equitable*** k***-coloring*** of a graph G is a proper k-coloring of G such that the sizes of the color classes differ by at most one.

- Note that for an equitable k-coloring f of a graph G, $\lfloor |V(G)|/k \rfloor \leq |f^{-1}(c)| \leq \lceil |V(G)|/k \rceil$ for each color c.

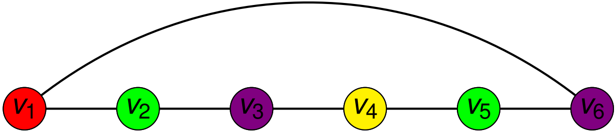


Figure: For $G = C_6$ above, we have: $|V(G)| = 6$, $k = 4$, $\lfloor |V(G)|/k \rfloor = 1$, and $\lceil |V(G)|/k \rceil = 2$.

Equitable Coloring

- An ***equitable k-coloring*** of a graph G is a proper k-coloring of G such that the sizes of the color classes differ by at most one.

- Note that for an equitable k-coloring f of a graph G, $\lfloor |V(G)|/k \rfloor \leq |f^{-1}(c)| \leq \lceil |V(G)|/k \rceil$ for each color c.

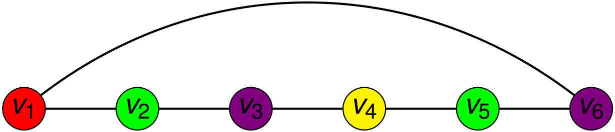


Figure: For $G = C_6$ above, we have: $|V(G)| = 6$, $k = 4$, $\lfloor |V(G)|/k \rfloor = 1$, and $\lceil |V(G)|/k \rceil = 2$.

- Intuitively, no color is overused or underused.

Application of Equitable Coloring

- In 1973, Tucker presented an application of equitable coloring: the garbage collection problem.

Application of Equitable Coloring

- In 1973, Tucker presented an application of equitable coloring: the garbage collection problem.
- Each vertex represents a garbage collection route, and routes that should not be run on the same day are connected by an edge.

Application of Equitable Coloring

- In 1973, Tucker presented an application of equitable coloring: the garbage collection problem.
- Each vertex represents a garbage collection route, and routes that should not be run on the same day are connected by an edge.
- The goal is to assign each vertex a color (day) so that about the same number of routes are run each day.

Application of Equitable Coloring

- In 1973, Tucker presented an application of equitable coloring: the garbage collection problem.
- Each vertex represents a garbage collection route, and routes that should not be run on the same day are connected by an edge.
- The goal is to assign each vertex a color (day) so that about the same number of routes are run each day.

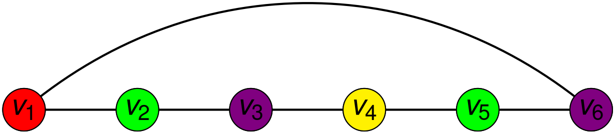


Figure: Each garbage collection route is assigned a day in which to run.

List Coloring

- A ***list assignment*** L for a graph G assigns each $v \in V(G)$ a list $L(v)$ of available colors.

List Coloring

- A ***list assignment*** L for a graph G assigns each $v \in V(G)$ a list $L(v)$ of available colors.

- The ***palette*** $\mathcal{L}$ of colors associated with L is $\cup_{v \in V(G)} L(v)$.

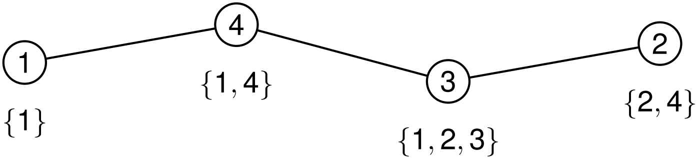


Figure: The palette of the list assignment for the copy of P_4 above is $\mathcal{L} = \{1, 2, 3, 4\}$.

List Coloring

- A ***list assignment*** L for a graph G assigns each $v \in V(G)$ a list $L(v)$ of available colors.

- The ***palette*** $\mathcal{L}$ of colors associated with L is $\cup_{v \in V(G)} L(v)$.

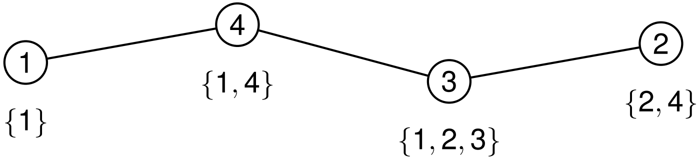


Figure: The palette of the list assignment for the copy of P_4 above is $\mathcal{L} = \{1, 2, 3, 4\}$.

- A ***proper L-coloring*** of G is a proper coloring f of G such that $f(v) \in L(v)$ for each $v \in V(G)$.

List Coloring Terminology

- If $|L(v)| = k$ for each $v \in V(G)$, then we say L is a k***-assignment*** for G.

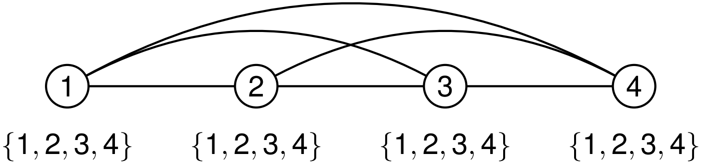
List Coloring Terminology

- If $|L(v)| = k$ for each $v \in V(G)$, then we say L is a k***-assignment*** for G.

- We say G is k***-choosable*** if a proper L-coloring of G exists whenever L is a k-assignment for G.

List Coloring Terminology

- If $|L(v)| = k$ for each $v \in V(G)$, then we say L is a k***-assignment*** for G.
- We say G is k***-choosable*** if a proper L-coloring of G exists whenever L is a k-assignment for G.
- For example, the complete graph K_n is n-choosable.



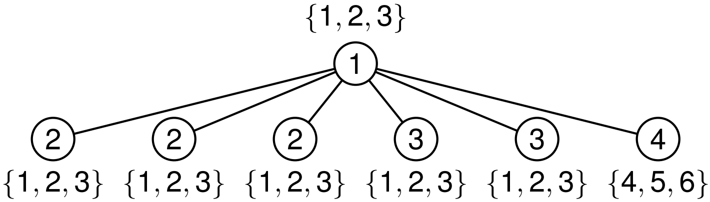
Equitable List Coloring

- In 2003, Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring called *equitable choosability*.

Equitable List Coloring

- In 2003, Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring called *equitable choosability*.

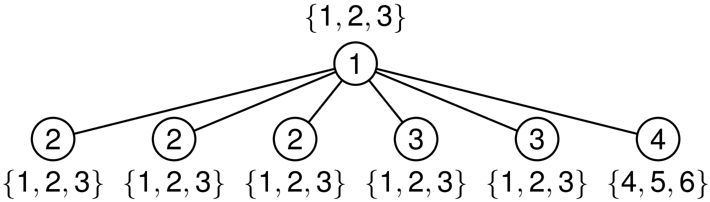
- For a k-assignment L, an ***equitable L-coloring*** f for a graph G is a proper L-coloring for G such that $|f^{-1}(c)| \leq \lceil |V(G)|/k \rceil$ for each $c \in \mathcal{L}$.



Equitable List Coloring

- In 2003, Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring called *equitable choosability*.

- For a k-assignment L, an ***equitable L-coloring*** f for a graph G is a proper L-coloring for G such that $|f^{-1}(c)| \leq \lceil |V(G)|/k \rceil$ for each $c \in \mathcal{L}$.



- Unlike equitable coloring, our only concern in equitable choosability is not overusing any color.

Proportional Choosability

- Recently, a new type of equitable list coloring called *proportional choosability* was introduced.

Proportional Choosability

- Recently, a new type of equitable list coloring called *proportional choosability* was introduced.
- For a graph G and a k-assignment L for G, the ***multiplicity*** of a color c, denoted by $\eta_L(c)$, is the number of vertices $v \in V(G)$ for which $c \in L(v)$.

Proportional Choosability

- Recently, a new type of equitable list coloring called *proportional choosability* was introduced.

- For a graph G and a k-assignment L for G, the ***multiplicity*** of a color c, denoted by $\eta_L(c)$, is the number of vertices $v \in V(G)$ for which $c \in L(v)$.

- For a k-assignment L, a ***proportional L-coloring*** for a graph G is a proper L-coloring for G such that $\lfloor \eta(c)/k \rfloor \leq |f^{-1}(c)| \leq \lceil \eta(c)/k \rceil$ for each color $c \in \mathcal{L}$.

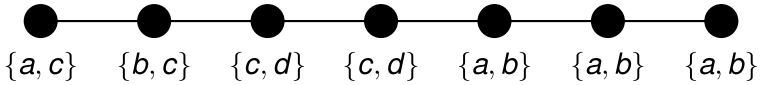


Figure: $\eta(a) = \eta(b) = \eta(c) = 4$ and $\eta(d) = 2$, so we must use a, b, and c exactly twice each and d exactly once.

Terminology

- If there exists a proportional L-coloring for G, then we say G is ***proportionally*** L***-colorable***.

Terminology

- If there exists a proportional L-coloring for G, then we say G is ***proportionally L-colorable***.

- G is ***proportionally k-choosable*** if G is proportionally L-colorable whenever L is a k-assignment for G.

Terminology

- If there exists a proportional L-coloring for G, then we say G is ***proportionally L-colorable***.
- G is ***proportionally k-choosable*** if G is proportionally L-colorable whenever L is a k-assignment for G.
- One application is assigning referee crews for an elimination-style basketball tournament, given that no crew may referee two games in a row.

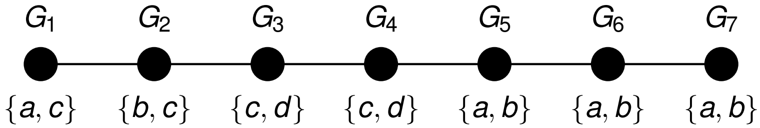


Figure: $\eta(a) = \eta(b) = \eta(c) = 4$ and $\eta(d) = 2$, so we must use a, b, and c exactly twice each and d exactly once.

Monotonicity Results

Theorem (Kaul et al. (2019))

If G is proportionally k-choosable, then G is proportionally $(k+1)$-choosable.

Monotonicity Results

Theorem (Kaul et al. (2019))

If G is proportionally k-choosable, then G is proportionally $(k+1)$-choosable.

Theorem (Kaul et al. (2019))

Suppose H is a subgraph of G. If G is proportionally k-choosable, then H is proportionally k-choosable.

Monotonicity Results

Theorem (Kaul et al. (2019))

If G is proportionally k-choosable, then G is proportionally $(k+1)$-choosable.

Theorem (Kaul et al. (2019))

Suppose H is a subgraph of G. If G is proportionally k-choosable, then H is proportionally k-choosable.

- Notice that these properties do not hold for equitable coloring and equitable list coloring.

Bounded Palette

- If $|L(v)| = k$ for each $v \in V(G)$ and $\mathcal{L} \subseteq \{1, \ldots, \ell\}$, then we say L is a (k, ℓ)-***assignment*** for G.

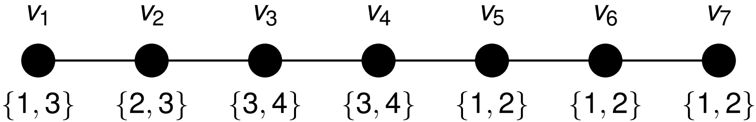


Figure: The palette is $\mathcal{L} = \{1, 2, 3, 4\}$ and $|L(v)| = 2$ for each $v \in V(G)$, so the list assignment above is a $(2, 4)$-assignment for $G = P_7$.

Bounded Palette

- If $|L(v)| = k$ for each $v \in V(G)$ and $\mathcal{L} \subseteq \{1, \ldots, \ell\}$, then we say L is a (k, ℓ)***-assignment*** for G.

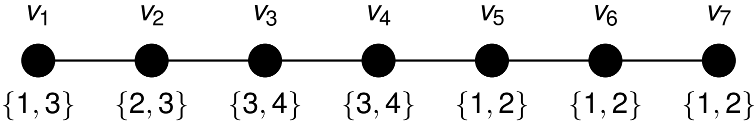


Figure: The palette is $\mathcal{L} = \{1, 2, 3, 4\}$ and $|L(v)| = 2$ for each $v \in V(G)$, so the list assignment above is a $(2, 4)$-assignment for $G = P_7$.

- We say G is ***proportionally*** (k, ℓ)***-choosable*** if G is proportionally L-colorable whenever L is a (k, ℓ)-assignment for G.

Starting Point

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

For each $k \in \mathbb{N}$, G is proportionally (k, k)-choosable if and only if G is equitably k-colorable.

Starting Point

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

For each $k \in \mathbb{N}$, G is proportionally (k, k)-choosable if and only if G is equitably k-colorable.

- This implies that G is proportionally $(2, 2)$-choosable if and only if G is equitably 2-colorable.

Starting Point

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

For each $k \in \mathbb{N}$, G is proportionally (k, k)-choosable if and only if G is equitably k-colorable.

- This implies that G is proportionally $(2, 2)$-choosable if and only if G is equitably 2-colorable.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

G is proportionally $(2, 2)$-choosable if and only if G is a bipartite graph with a bipartition X, Y satisfying $||X| - |Y|| \leq 1$.

Starting Point

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

Starting Point

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

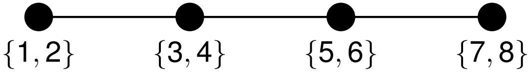
- Notice that if G is an n-vertex graph, then G is proportionally 2-choosable if and only if G is proportionally $(2, 2n)$-choosable.

Starting Point

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

- Notice that if G is an n-vertex graph, then G is proportionally 2-choosable if and only if G is proportionally $(2, 2n)$-choosable.

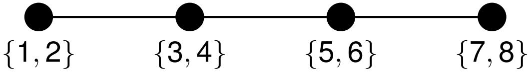


Starting Point

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

- Notice that if G is an n-vertex graph, then G is proportionally 2-choosable if and only if G is proportionally $(2, 2n)$-choosable.



- **Question:** Is there a constant μ such that any graph G is proportionally 2-choosable if and only if G is proportionally $(2, \mu)$-choosable?

Questions Guiding Our Research

- **Question:** Is there a constant μ such that any graph G is proportionally 2-choosable if and only if G is proportionally $(2, \mu)$-choosable?

Questions Guiding Our Research

- **Question:** Is there a constant μ such that any graph G is proportionally 2-choosable if and only if G is proportionally $(2, \mu)$-choosable?

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is proportionally $(k, \ell + 1)$*-choosable, then G is proportionally* (k, ℓ)*-choosable.*

Questions Guiding Our Research

- **Question:** Is there a constant μ such that any graph G is proportionally 2-choosable if and only if G is proportionally $(2, \mu)$-choosable?

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is proportionally $(k, \ell + 1)$-choosable, then G is proportionally (k, ℓ)-choosable.

- **Question:** For each $\ell \geq 2$, what graphs are proportionally $(2, \ell)$-choosable?

Questions Guiding Our Research

- **Question:** Is there a constant μ such that any graph G is proportionally 2-choosable if and only if G is proportionally $(2, \mu)$-choosable?

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is proportionally $(k, \ell + 1)$-choosable, then G is proportionally (k, ℓ)-choosable.

- **Question:** For each $\ell \geq 2$, what graphs are proportionally $(2, \ell)$-choosable?

- Notice that if $i \geq 2$ and $\mathcal{G}_i$ is the set of graphs that are proportionally $(2, i)$-choosable, then $\mathcal{G}_2 \supseteq \mathcal{G}_3 \supseteq \mathcal{G}_4 \supseteq \dots$

Summary of Results

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A graph G is proportionally 2-choosable if and only if G is proportionally $(2, \ell)$-choosable for $\ell \geq 5$.

Summary of Results

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A graph G is proportionally 2-choosable if and only if G is proportionally $(2, \ell)$*-choosable for* $\ell \geq 5$.

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A connected graph G is proportionally $(2, 4)$*-choosable if and only if* $G = P_n$ *where* $n \leq 5$ *or* $n = 7$.

Summary of Results

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A graph G is proportionally 2-choosable if and only if G is proportionally $(2, \ell)$-choosable for $\ell \geq 5$.

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A connected graph G is proportionally $(2, 4)$-choosable if and only if $G = P_n$ where $n \leq 5$ or $n = 7$.

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A connected graph G is proportionally $(2, 3)$-choosable if and only if $G = P_n$ for some $n \in \mathbb{N}$.

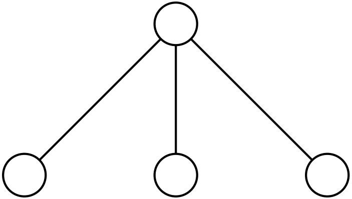
A First Result

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

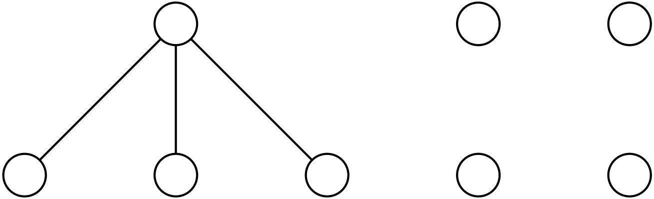
If G contains a copy of $K_{1,3}$ as a subgraph, then G is not proportionally $(2,3)$-choosable. Consequently, if a graph G is proportionally $(2,\ell)$-choosable for some $\ell \geq 3$, then $\Delta(G) \leq 2$.

Proof Idea

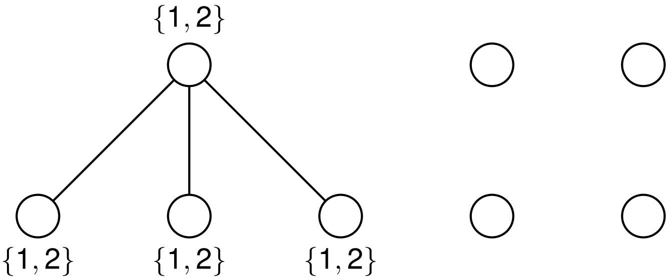
Proof Idea



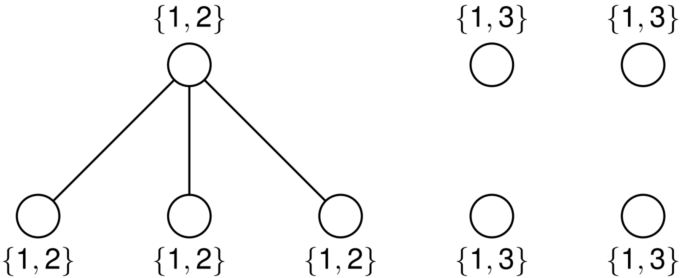
Proof Idea



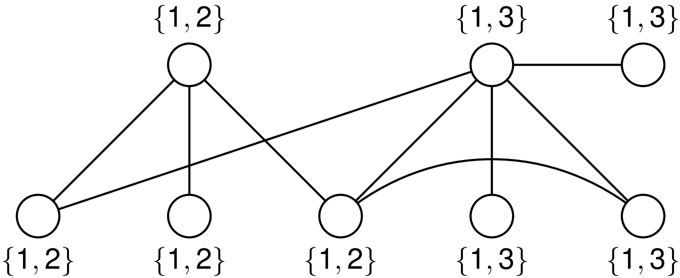
Proof Idea



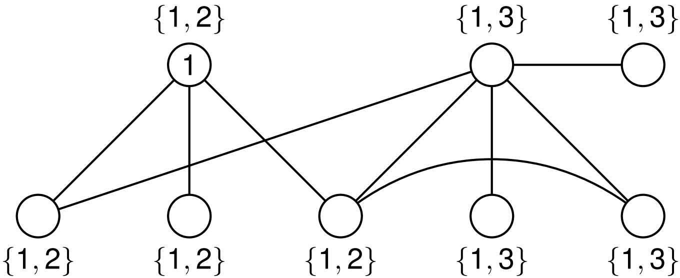
Proof Idea



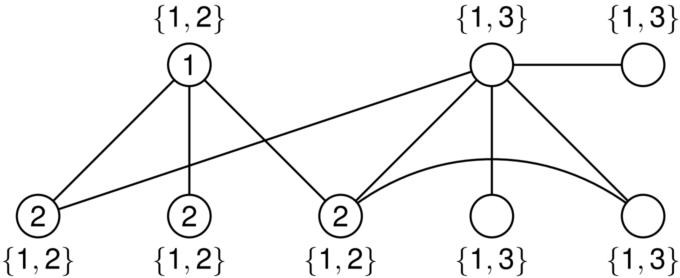
Proof Idea



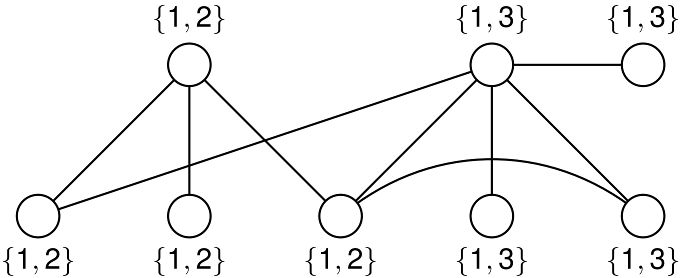
Proof Idea



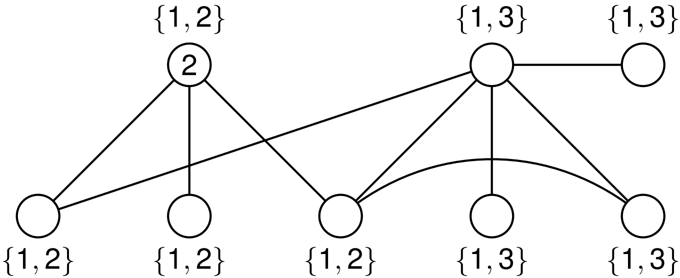
Proof Idea



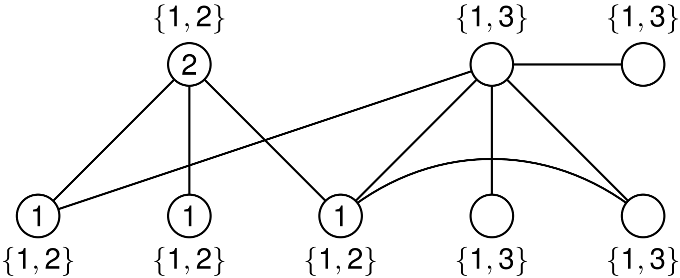
Proof Idea



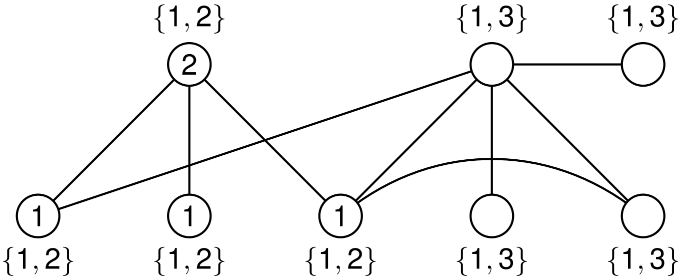
Proof Idea



Proof Idea



Proof Idea



- Thus, a graph which contains $K_{1,3}$ is not proportionally $(2,3)$-choosable.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If a graph G is proportionally $(2, \ell)$-choosable for some $\ell \geq 3$, then $\Delta(G) \leq 2$.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If a graph G is proportionally $(2, \ell)$-choosable for some $\ell \geq 3$, then $\Delta(G) \leq 2$.

- From this result, we know that if a connected graph G is proportionally $(2, \ell)$-choosable for $\ell \geq 3$, then G is either a path or a cycle.

Further Results

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If $G = C_n$ for some $n \geq 3$, then G is not proportionally $(2, 3)$-choosable.

Further Results

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

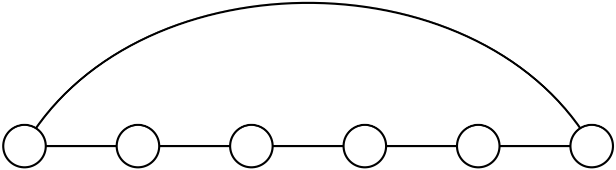
If $G = C_n$ for some $n \geq 3$, then G is not proportionally $(2, 3)$-choosable.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

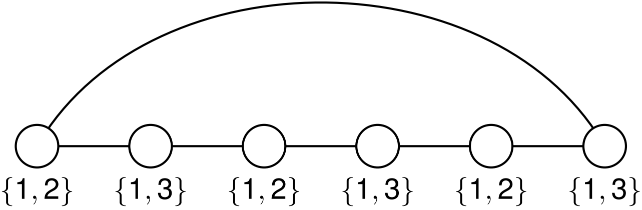
If a graph contains a cycle, then it is not proportionally $(2, \ell)$-choosable for each $\ell \geq 4$.

Proof Idea

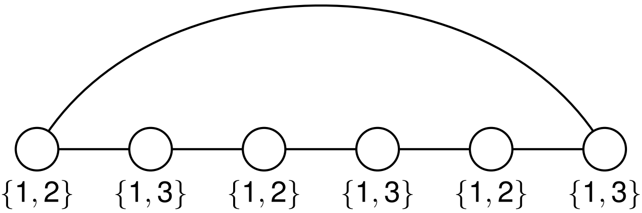
Proof Idea



Proof Idea

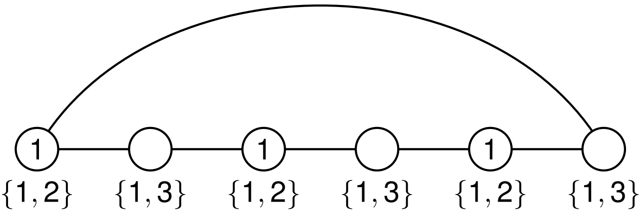


Proof Idea



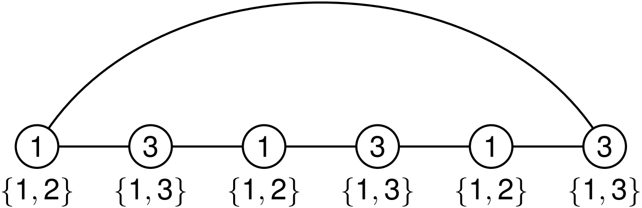
- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.

Proof Idea



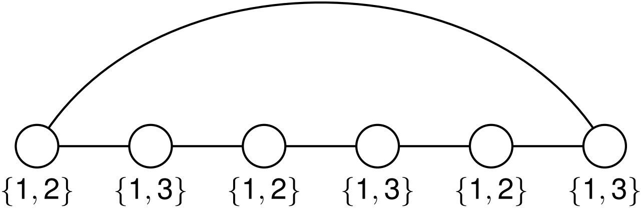
- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.

Proof Idea



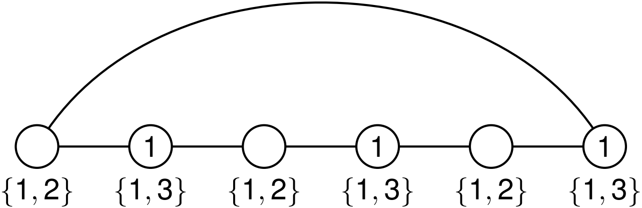
- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.

Proof Idea



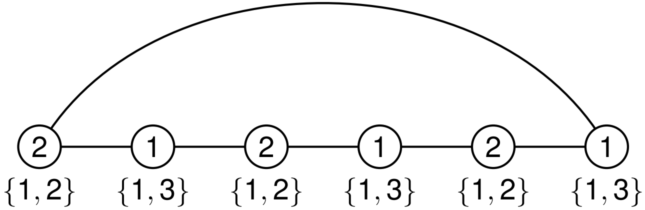
- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.

Proof Idea



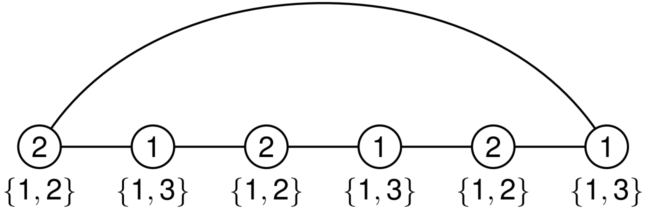
- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.

Proof Idea



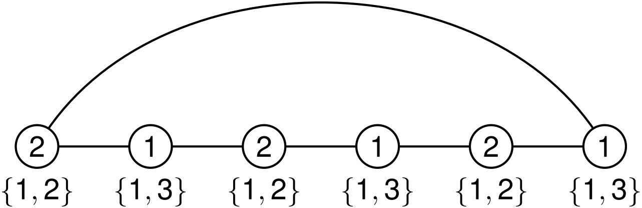
- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.

Proof Idea

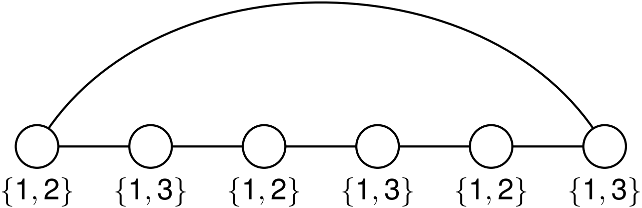


- Notice that $\eta(1) = 6$; thus, we must use 1 exactly three times.
- This implies that 2 is either underused or overused, so G is not proportionally L-colorable.

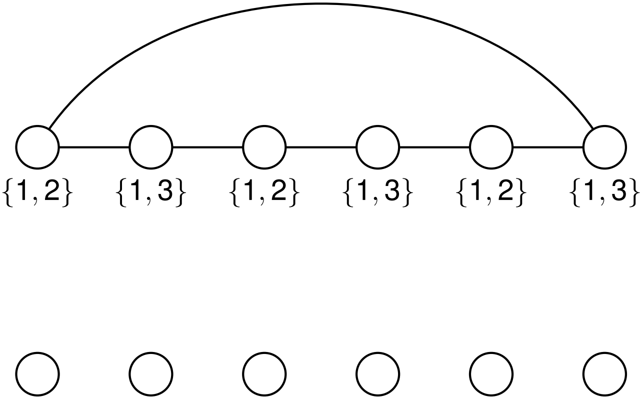
Proof Idea



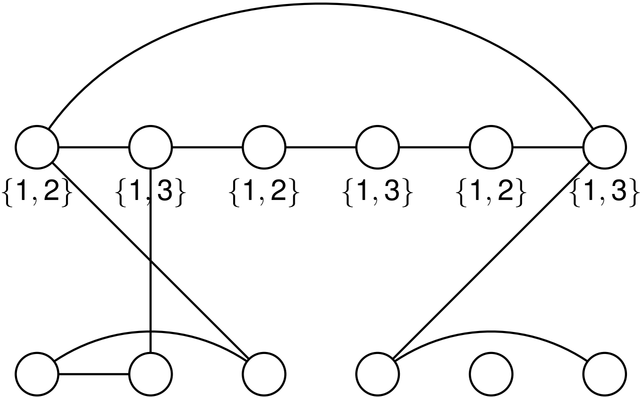
Proof Idea



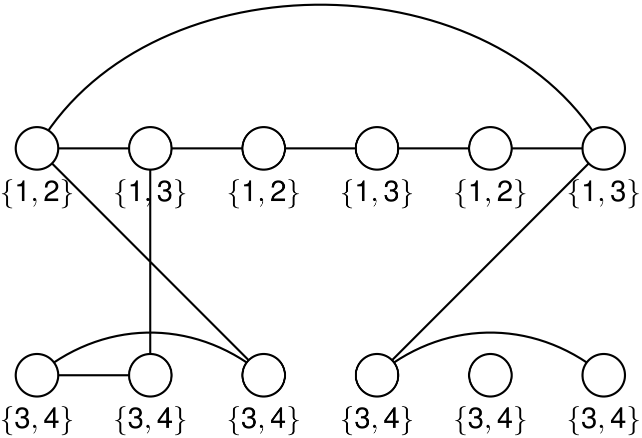
Proof Idea



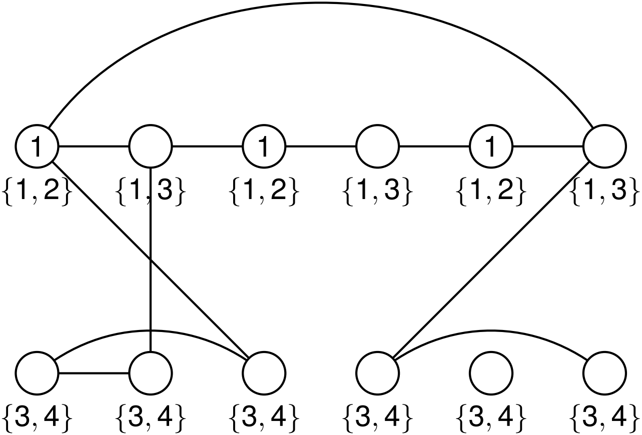
Proof Idea



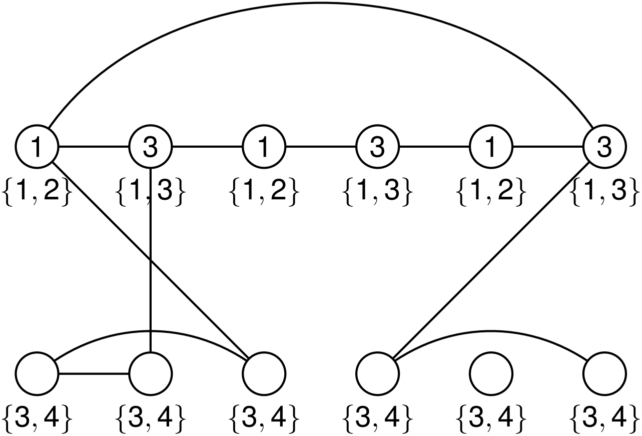
Proof Idea



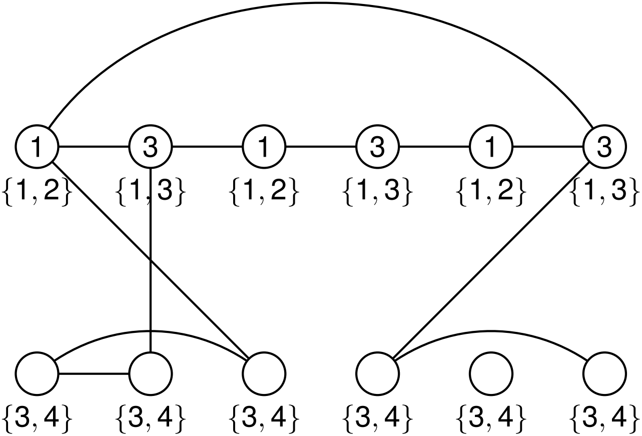
Proof Idea



Proof Idea

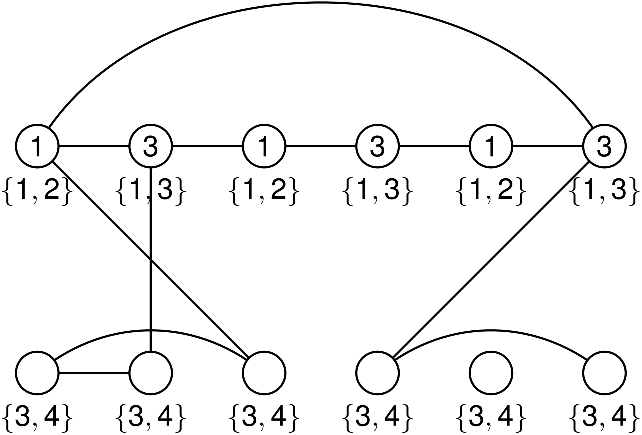


Proof Idea

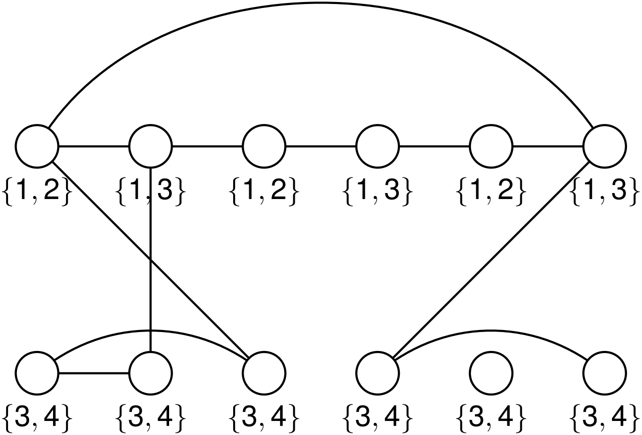


- Notice that the color 2 is underused.

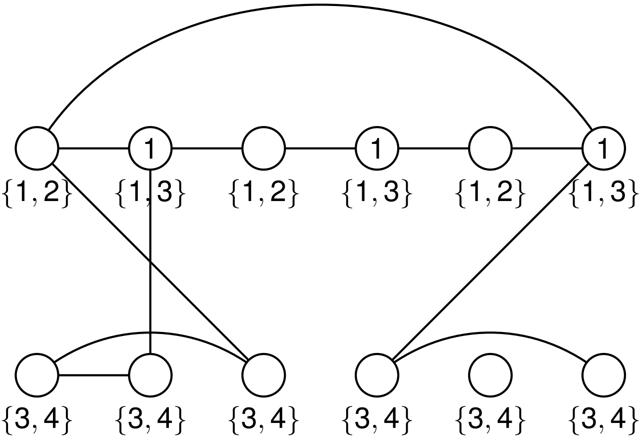
Proof Idea



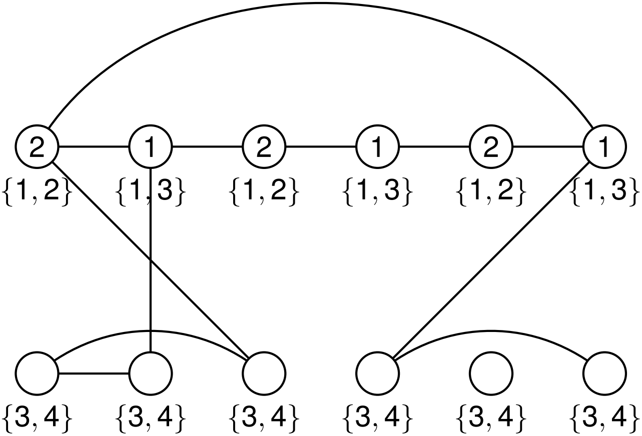
Proof Idea



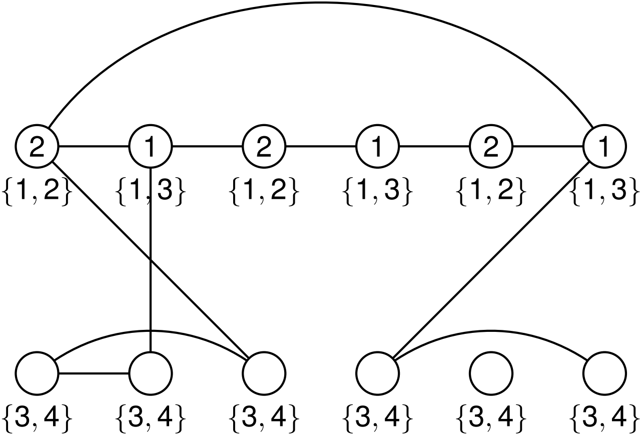
Proof Idea



Proof Idea



Proof Idea



- Here, 2 is overused; thus, any graph that contains C_n is not proportionally $(2, 4)$-choosable.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If $G = C_n$ for some $n \geq 3$, then G is not proportionally $(2, 3)$-choosable.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If $G = C_n$ for some $n \geq 3$, then G is not proportionally $(2, 3)$-choosable.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If a graph contains a cycle, then it is not proportionally $(2, \ell)$-choosable for each $\ell \geq 4$.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If $G = C_n$ for some $n \geq 3$, then G is not proportionally $(2, 3)$-choosable.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If a graph contains a cycle, then it is not proportionally $(2, \ell)$-choosable for each $\ell \geq 4$.

- The first result implies that if a connected graph G is proportionally $(2, \ell)$-choosable for $\ell \geq 3$, then $G = P_n$ for some $n \in \mathbb{N}$.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

Suppose $n \in \mathbb{N}$. If $G = P_n$, then G is proportionally $(2, 3)$-choosable.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

Suppose $n \in \mathbb{N}$. If $G = P_n$, then G is proportionally $(2, 3)$-choosable.

- From this result, we can obtain our second characterization.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

Suppose $n \in \mathbb{N}$. If $G = P_n$, then G is proportionally $(2, 3)$-choosable.

- From this result, we can obtain our second characterization.

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A connected graph G is proportionally $(2, 3)$-choosable if and only if $G = P_n$ for some $n \in \mathbb{N}$.

Another Result

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If a graph contains $P_3 + P_3$, then it is not proportionally $(2, \ell)$-choosable for each $\ell \geq 5$.

Another Result

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

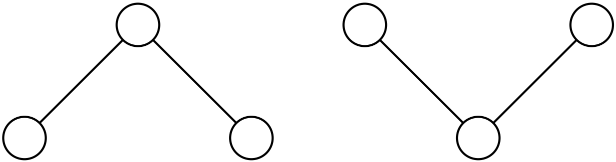
If a graph contains $P_3 + P_3$, then it is not proportionally $(2, \ell)$-choosable for each $\ell \geq 5$.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

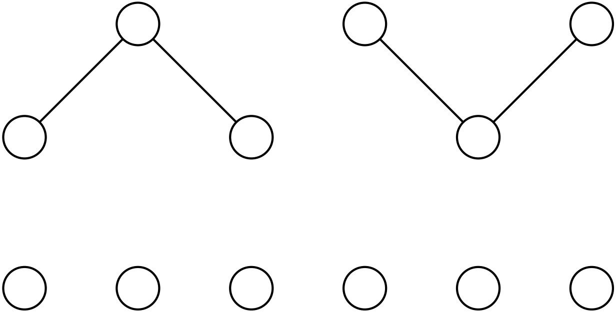
P_n is not proportionally $(2, 4)$-choosable for $n = 6$ and for each $n \geq 8$.

Proof Idea

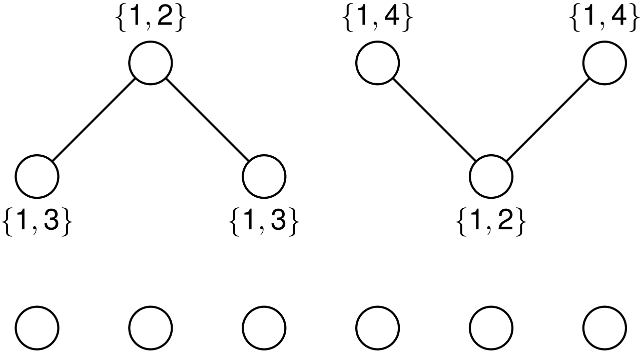
Proof Idea



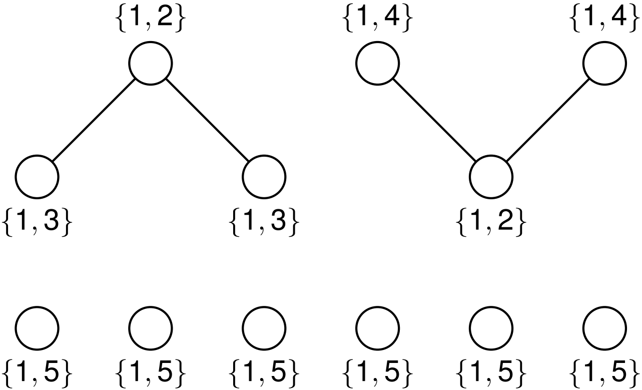
Proof Idea



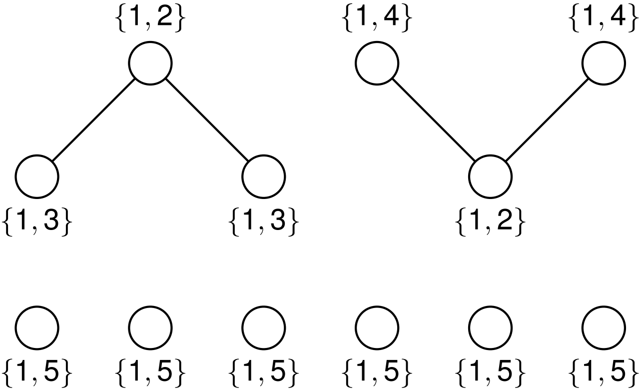
Proof Idea



Proof Idea

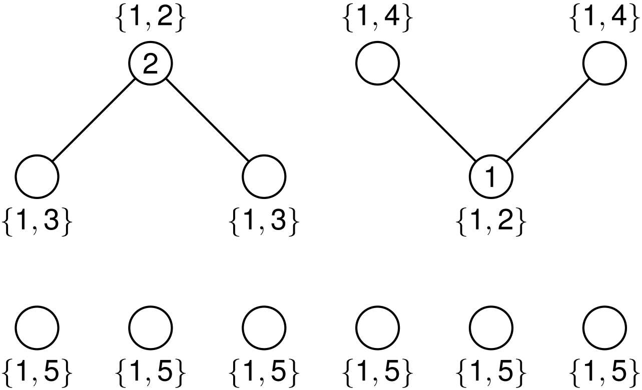


Proof Idea



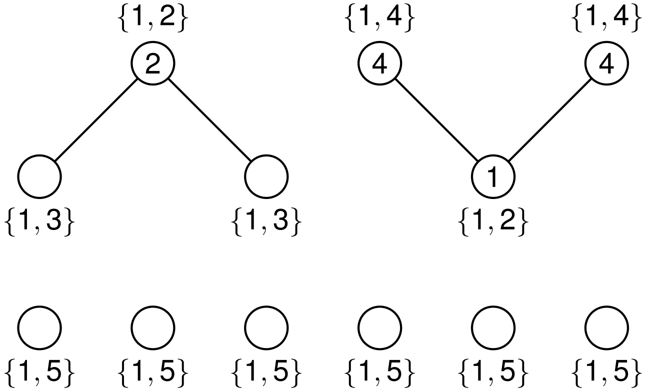
- Notice that $\eta(2) = 2$, so we must use 2 exactly once.

Proof Idea



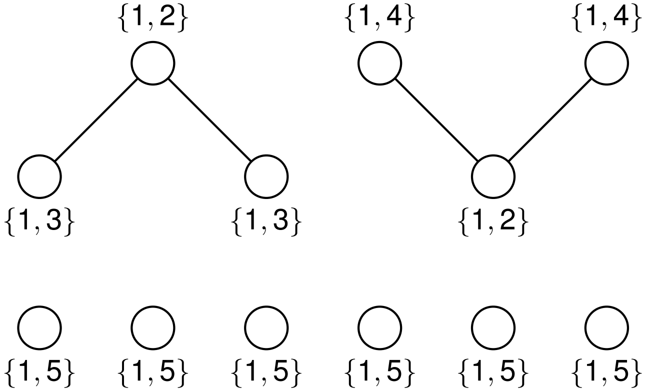
- Notice that $\eta(2) = 2$, so we must use 2 exactly once.

Proof Idea



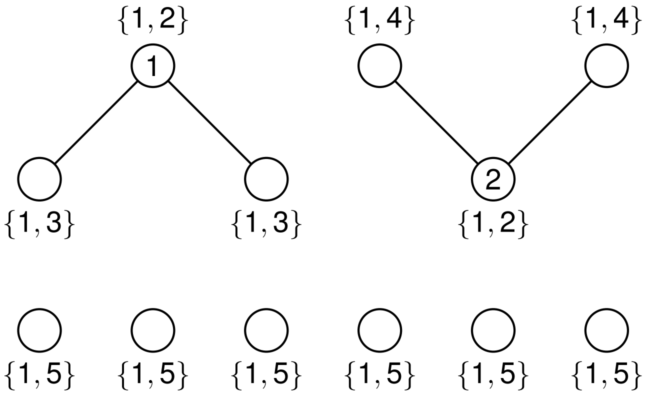
- Notice that $\eta(2) = 2$, so we must use 2 exactly once.

Proof Idea



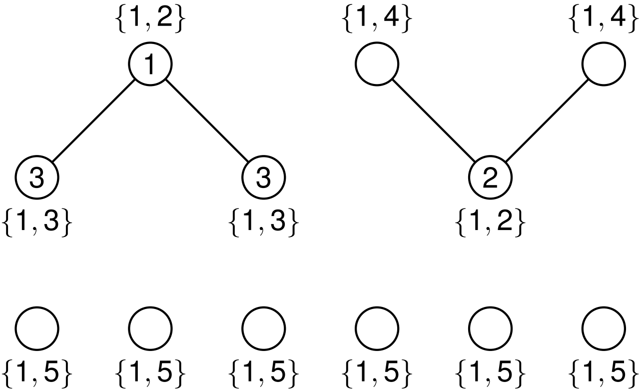
- Notice that $\eta(2) = 2$, so we must use 2 exactly once.

Proof Idea



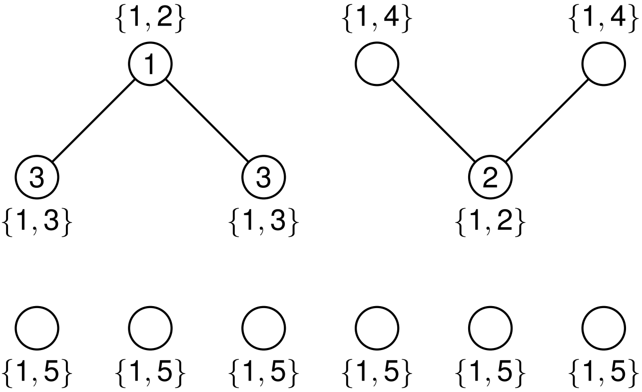
- Notice that $\eta(2) = 2$, so we must use 2 exactly once.

Proof Idea



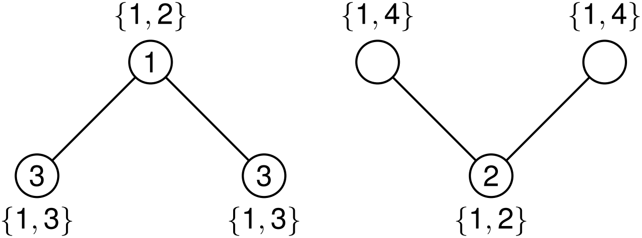
- Notice that $\eta(2) = 2$, so we must use 2 exactly once.

Proof Idea

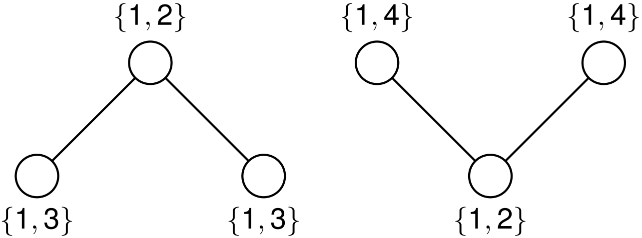


- Notice that $\eta(2) = 2$, so we must use 2 exactly once.
- Thus, any graph that contains $P_3 + P_3$ is not proportionally $(2, 5)$-choosable.

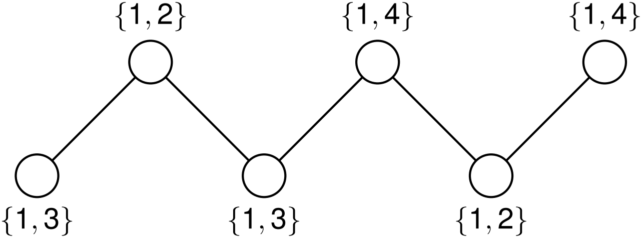
Proof Idea



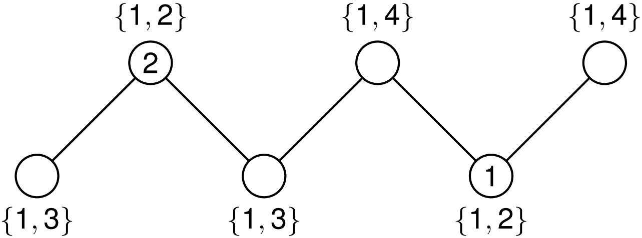
Proof Idea



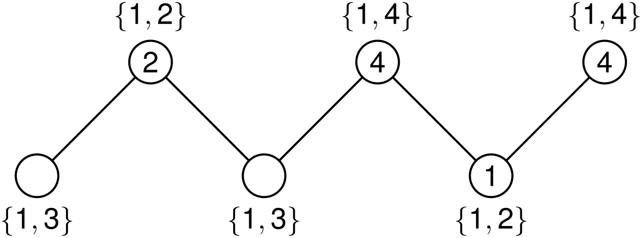
Proof Idea



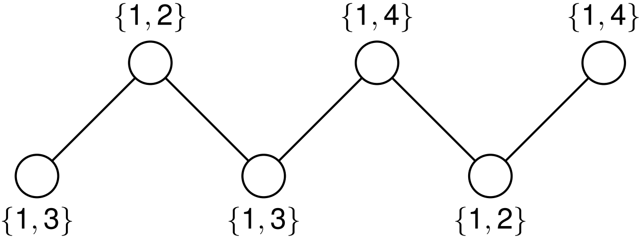
Proof Idea



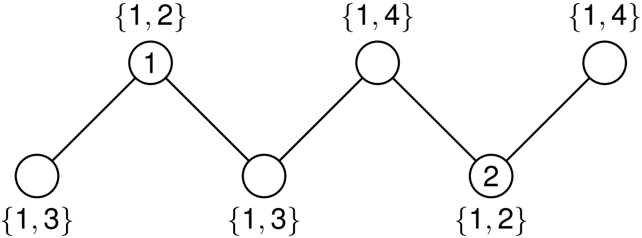
Proof Idea



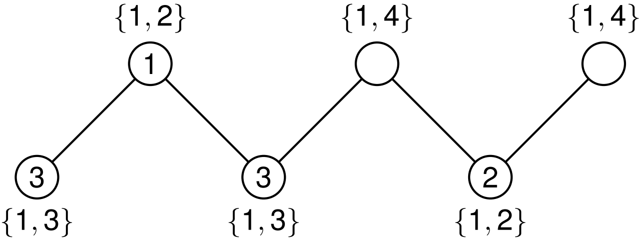
Proof Idea



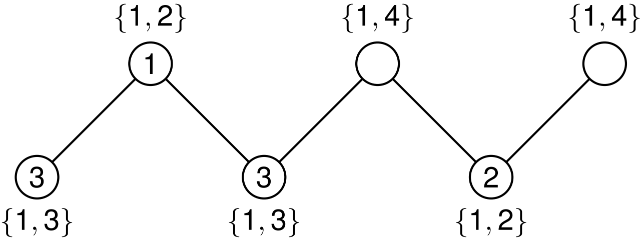
Proof Idea



Proof Idea



Proof Idea



- Thus, P_6 is not proportionally $(2, 4)$-choosable.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is not proportionally $(2, \ell)$-choosable where $\ell \geq 2$, then $G + P_2$ is not proportionally $(2, \ell)$-choosable.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is not proportionally $(2, \ell)$-choosable where $\ell \geq 2$, then $G + P_2$ is not proportionally $(2, \ell)$-choosable.

- This means $P_6 + P_2$, and thus P_8, is not proportionally $(2, 4)$-choosable. It turns out the same is true for P_9.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is not proportionally $(2, \ell)$-choosable where $\ell \geq 2$, then $G + P_2$ is not proportionally $(2, \ell)$-choosable.

- This means $P_6 + P_2$, and thus P_8, is not proportionally $(2, 4)$-choosable. It turns out the same is true for P_9.
- By induction, P_n is not proportionally $(2, 4)$-choosable for $n \geq 8$.

Conclusions

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If G is not proportionally $(2, \ell)$-choosable where $\ell \geq 2$, then $G + P_2$ is not proportionally $(2, \ell)$-choosable.

- This means $P_6 + P_2$, and thus P_8, is not proportionally $(2, 4)$-choosable. It turns out the same is true for P_9.
- By induction, P_n is not proportionally $(2, 4)$-choosable for $n \geq 8$.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

P_n is not proportionally $(2, 4)$-choosable for $n = 6$ and for each $n \geq 8$.

Conclusions

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

Conclusions

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A connected graph G is proportionally $(2, 4)$-choosable if and only if $G = P_n$ where $n \leq 5$ or $n = 7$.

Conclusions

Theorem (Kaul et al. (2019))

G is proportionally 2-choosable if and only if G is a linear forest such that the largest component of G has at most five vertices and all other components of G have two or fewer vertices.

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

A connected graph G is proportionally $(2, 4)$-choosable if and only if $G = P_n$ where $n \leq 5$ or $n = 7$.

- For now, we've used a computer-assisted proof to show that P_7 is proportionally $(2, 4)$-choosable.

Conclusions

- If a graph G is proportionally $(2, 5)$-choosable, then G is a forest.

Conclusions

- If a graph G is proportionally $(2, 5)$-choosable, then G is a forest.

- Furthermore, G must be a linear forest.

Conclusions

- If a graph G is proportionally $(2, 5)$-choosable, then G is a forest.
- Furthermore, G must be a linear forest.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

If a graph contains $P_3 + P_3$, then it is not proportionally $(2, \ell)$-choosable for each $\ell \geq 5$.

- This implies that if a graph contains P_6, then it is not proportionally $(2, 5)$-choosable.

Conclusions

- If a graph G is proportionally $(2,5)$-choosable, then G is a forest.
- Furthermore, G must be a linear forest.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

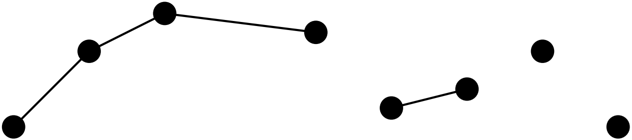
If a graph contains $P_3 + P_3$, then it is not proportionally $(2,\ell)$-choosable for each $\ell \geq 5$.

- This implies that if a graph contains P_6, then it is not proportionally $(2,5)$-choosable.
- Also, no two components may contain P_3.

Conclusions

Theorem (Mudrock, Piechota, S., and Wagstrom (2019))

For each $\ell \geq 5$, a graph G is proportionally $(2, \ell)$-choosable if and only if G is a linear forest such that the largest component of G has at most 5 vertices and all other components of G have at most 2 vertices.



An Open Question

- **Open Question:** For $\ell = 3, 4$, what graphs are proportionally $(2, \ell)$-choosable?

An Open Question

- **Open Question:** For $\ell = 3, 4$, what graphs are proportionally $(2, \ell)$-choosable?

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

$P_6 + P_1$ is proportionally $(2, 4)$-choosable.

An Open Question

- **Open Question:** For $\ell = 3, 4$, what graphs are proportionally $(2, \ell)$-choosable?

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

$P_6 + P_1$ *is proportionally* $(2, 4)$*-choosable.*

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

$C_4 + P_1$ *is proportionally* $(2, 3)$*-choosable.*

An Open Question

- **Open Question:** For $\ell = 3, 4$, what graphs are proportionally $(2, \ell)$-choosable?

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

$P_6 + P_1$ is proportionally $(2, 4)$-choosable.

Proposition (Mudrock, Piechota, S., and Wagstrom (2019))

$C_4 + P_1$ is proportionally $(2, 3)$-choosable.

- Another possible area of research is the proportional choosability of graphs with a bounded palette for lists of size other than two.

Thank You!

- Questions?